# On the Optimal Quantity of Public Goods and Some Related Issues

Tamunopriye J. Agiobenebo, Ph.D.*

tjagiobenebo@yahoo.com

Department of Economics, University of Port Harcourt, Port Harcourt, Nigeria

and

Department of Economics, University of Botswana, Gaborone, Botswana

*The author is a Professor of Economics at the University of Port Harcourt, Port Harcourt, Nigeria and currently a Visiting Professor of Economics at the University of Botswana, Gaborone, Botswana. The initial draft of this paper benefited from insightful comments by Professor Ismail Cole and the participants in the Departmental Seminar at the University of Botswana, where the initial draft was presented for discussion. This edition of the paper has also benefited from comments from Professor Conley.

# On the Optimal Quantity of Public Goods and Related Issues

Tamunopriye J. Agiobenebo, Ph.D.

**ABSTRACT** This paper seeks to reopen a discussion that the profession has considered settled and closed, namely, the issue of the optimal quantity of a public good to supply. **Its focus is on the determination of the optimal quantity to supply of a public good in the Pigovian model as popularized by Musgrave**. It argues that the vertical summation of the individual demand curves in the Pigovian model is as inappropriate as the rejected horizontal summation of individuals' consumption of public goods. The horizontal summation is inconsistent with the physical realities of public good supply suggesting an m-multiple of the quantity that is actually available, which is an illusion. The vertical summation while having the advantage of informing on efficiency taxes and equitable cost sharing formula suffers from the fallacy of aggregation and exaggerates the aggregate demand for public goods, thus misleading supply decisions.

This realization comes from reckoning with the basic properties of pure public goods in particular nonrivalry – the joint supply property. Given this property, the paper submits that **the optimal quantity of a public good is the largest quantity demanded by any single consumer** (individually or as a collective). A corollary of this is that public goods consumption is not validly subject to aggregation by any means. Aggregation is irrelevant and that individual demand curves or schedules are required only for the determination of optimal benefit taxes and equitable cost sharing formula. That is, the individual demand curves for a public good or service should be considered **only for the purpose of determining each person's fair and equitable share of the cost of supply** (i.e. based on the individuals' marginal valuations) as shown in Figure 4. In other words, the so-called (collective) willingness to pay curve is only confusing issues and hence not required. This in no way implies the consumption of separate quantities to be added up. Rather, it stages the consumption of the same quantity by all with different payments that are added up to finance the supply, which conforms to the Pigovian solution and indeed all the solutions that have been advanced. Yet, it is unique from the earlier solutions.

An equitable cost sharing formula that guarantees an efficient financing scheme is also proposed. It is a benefit share weighted cost sharing formula, which obviates the potential threat of fiscal drag. It is found that barring information failure, the public budget should ordinarily be enjoying surplus under optimal benefit taxes. That is with optimal benefit taxes, balanced budget is within easy reach with relief for taxpayers in relation to service benefits enjoyed and therefore, less resentment to taxation could be anticipated.

These observations and findings need to be given serious thoughts. Though, the rationale for government intervention in an otherwise market economy is primarily seen in market failure, the ultimate justification is the gain in welfare from the achievement of Pareto improvements in resource allocation. It follows that the possibility of further Pareto improvements in welfare must be a welcome development. JEL: H41

**Key words**: Optimal public good supply; Pigovian model; irrelevance of vertical summation; the fallacy of aggregation; efficient financing; efficiency taxes; optimal benefit taxes; and equitable cost sharing formula.

# Introduction

Public goods are commodities that exhibit certain properties, namely, nonrivalry, non-excludability, non-exhaustion and even indivisibility in consumption. These properties are not altogether mutually exclusive and the other three seem to contribute to nonrivalry. The focus of this paper, however, is the implications of nonrivalry as the key defining characteristic of a public good with optimal quantity in view. Nonrivalry means that the consumption of any user of a public good does not reduce the amount available for others. Thus, the non-rivalrous property does imply the non-exhaustion property subject only to crowding or congestion, which reduces the quality of what is available to all. Buchanan (1968) seems to suggest that distance from facility locations can also cause variations in quality. Quality variations, however, would be reflected in the marginal valuations of the users. Crowding or congestion, in any case, could be controlled in principle.

It is important to recognize that non-rivalrous consumption implies that making a public good available to any one single individual makes it possible to provide it for everyone without additional cost. This is the joint supply property. Thus, joint consumption means that public goods are used simultaneously by all consumers without individual exclusion. It is for this reason that public goods are also called "collective goods" and even "non-exchange goods". It follows that the additional cost of providing an existing public good for an additional user is zero. This is clearly exemplified by Buchanan's example of mosquito repelling by Tizio and Caio, which exhibits the property of zero additional cost. Further, it clearly reveals the waste in the competitive supply of mosquito repelling by Tizio and Caio. As he rightly noted, the death of one mosquito benefits each man simultaneously. So, once the mosquitoes are dead any number of migrants can join the community and enjoy its peace and health without additional cost. After all, no mosquitoes die twice or do they?

This should not be confused with providing two, three or any number of a public facility be it a baseball or football stadium or highway, which involve space and or time dimensions and quite different considerations. Building any number of a public facility is perhaps best analyzed within a multi-plant paradigm than within the context of marginal adjustment. The focus here, however, is on what it costs to admit a new user for an existing facility? This motivates a number of questions. What is the equilibrium condition of the new user?[1] Or is this entry irrelevant to the optimal solution? If it is relevant then, what is the optimal quantity of a public good to supply? Further, what is the efficient means for financing the provision? In addition, what is the equitable cost sharing formula? These questions are intimately related. And they have attracted the attention of some of the best minds and known names in the profession. Attempts to answer the question on the optimal quantity to supply have faced a major challenge i.e. the challenge of discovering a costless means of truthful preference revelation, aggregation and interpretation. It led to economics incursions into political analysis. Many, if not all, of the economic theories of politics, directly or indirectly speak to these issues.

Wicksell among others had the insight that political institutions and procedures might be useful means of preference revelation and for the aggregation of preferences, he recommended unanimity. Unanimity is costlessly obtainable in a homogeneous population or society. Wicksell is right that if public choices can be made by unanimity, they will both be efficient and equitable. Most unfortunately, homogeneous societies or populations hardly, if ever, exist. Conceivably, unanimity could also be obtained if the benefits and burdens of public choices, decisions and actions are distributed in such a manner as to equate marginal benefits (valuation) to marginal costs (taxes) for each and every member of society. This condition decried by its logistic requirements is most unlikely sounding. Under conditions of heterogeneity, preference diversity escalates in the powers of $2^m$, where m is the number of individuals or homogeneous subgroups in the society. The result is multiple suggestions and the “best” for society becomes controversial. Since all the conditions that favour unanimity are easily violated in the real world, political practice conveniently settles for some majority voting rules, which unfortunately are known not to produce transitive public choices. This is the paradox of majority rule in the literature. The pertinent question, therefore, is what are

[1] It is recognized that in time or eventually crowding will lead to expansion but this is a scale problem rather than a marginal adjustment.

the alternatives for optimal public decision-making?

Many suggestions litter the field and there is no unique solution. As for truthful preference revelation, there seem to be no better alternative to the political process as imperfect as it is, though economists may suggest survey approach. For preference aggregation, economists have suggested the cardinal utility approach, which is conceptually elegant but not a practicable proposition founded on an unobservable quantity – marginal utility. Others include Pareto superiority, Pareto improvement and Pareto optimality that exhibit varying degrees of practicality and the economic surplus technique with reasonable degree of practicality though with a lot of limitations. On the equity axis, the sacrifice rules and compensation principles have been advanced with little or no possibility of rational implementation. Admittedly, the compensation principle is actually being practised but with a lot of imperfections. Consequently, many questions remain open.

## The Voluntary Exchange Theory of Optimal Supply Public Goods

### The Demand for Public Goods

Given the property of joint consumption, consumers need not reveal their true demand for public goods in a marketplace. This means that there are no demand curves for public goods. This is well known to the profession. Yet, it is the submission of this paper that contrary to the conventional wisdom, this difficulty does not pose any problems for the determination of the optimal quantity of a public good. Even when, society is made up of honest citizens, the vertical aggregation of the true demands of the consumers does not reveal the optimal quantity but overestimates it. This is the basic thesis of this paper.

### The Pigovian Approach

Pigou (1928) provided an analysis of the efficient allocation of resources to public and private goods. He used the utilitarian approach assuming cardinal utility for which he was criticized severely. Yet, the profession is grateful for the insights he provided on many of the pertinent questions. Private commodities are divisible and are provided with individual exclusion. Divisibility and exclusion imply rival consumption and exhaustion. Thus, for private goods, $q_{pi}$ is not necessarily equal to $q_{pj}$, $i \neq j$; $q_{pi} \leq Q_{ps}$ (where $Q_{ps}$ is the total supply of the private good) such that $\sum_1^m q_{pi}$ (aggregate consumption) $\leq Q_{ps}$. The strict equality is the market clearing condition and the interpretation of the exhaustion property, while $\sum_1^m q_{pi}$ is the definition of horizontal summation of individual demands. Given these characteristics, the demand for private goods gets expressed in the marketplace and is satisfied by profit-seeking entrepreneurs.

It is quite different for public goods. Given joint consumption, consumers can free ride, get dishonest and falsify their preferences, since they could be consumed without paying for them. Thus, for public goods, we have

$q_{si} = q_{sj}$, $i \neq j$; i.e. $q_{s1} = q_{s2} = q_{s3} = \ldots = q_{sm} = Q_{ss}$ (total supply of the public good).

Hence, total consumption of public goods by horizontal summation leads to

$\sum_1^m q_{pi} = mQ_s = Q_s$, $m \geq 1$.

If the strict inequality holds, then, the horizontal aggregation is inconsistent with reality. It actually becomes an illusion. For this reason, it is rejected. This aggregation problem arise from the fact that a unit of public goods once supplied provides a multiplicity of consumption units, all of which are somehow identical, which is the meaning of (m > 1).[2] Consequently, it will not be efficient to exclude any one from its consumption. The foregoing discussion shows that the analysis of the efficient allocation of resources to public goods supply must proceed under restrictive assumptions and pseudo demand curves. This has been the case. Despite, its obvious deficiencies, the voluntary exchange approach is instructive for the insights it provides on many of the pertinent questions the theory of public goods supply has to answer.

For purposes of geometrical illustration, let m = 2. That is, the analysis assumes a two-person economy that produces and consumes private and public goods from here on. Figure 1 sets out the usual analysis of a private good (a la Musgrave 1958, 1989). The curves $D_{pA}$

---

[2] See Buchanan (1968), The Demand and Supply of Public Goods: Chapter 4.

and $D_{pA}$ denote the demand curves for individual A and B, respectively for the private good. The differences in the demand curves are presumed to reflect the differences in the socioeconomic circumstances and tastes of the consumers. The market demand curve $D_{pA+B}$ is derived by horizontal summation of the individual demand curves, i.e. $D_{pA+B} = (D_{pA} + D_{pB})$. Any point on the individual demand curves has the usual interpretation, i.e. the marginal valuation of the usefulness of the commodity at a given price to the consumer and hence a willingness to consume or pay for consumption by the consumer (ceteris paribus).

The market demand curve shows how much all individuals are willing to consume at the schedule of prices. Given the supply curve S, which is the schedule of marginal costs for supplying each quantity of output, the market forces of demand and supply determine the market clearing (equilibrium) price, $P_p^*$ in Figure 1 with the corresponding equilibrium quantity $Q_p^* = OQ_a + OQ_B$.

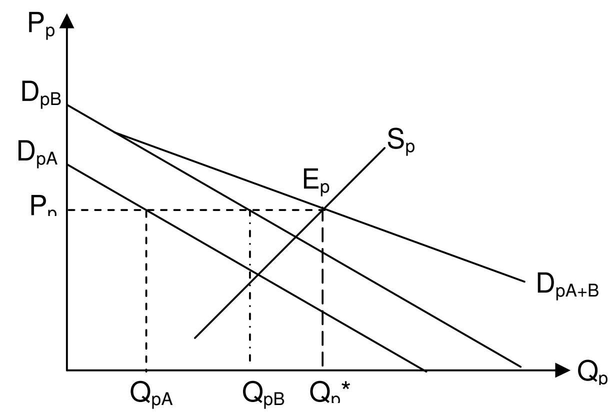


Figure 1: Analysis of the Optimal Supply of Private Good

Figure 1 shows that commodity $Q_p$ is divisible and individuals A and B given their socio-economic circumstances and preferences purchased different quantities, which equalized their marginal utilities from consuming the commodity in equilibrium at the same price. The divisibility, rival consumption, exhaustion and adding-up properties of the private good are all manifest in the diagram.

Figure 2 sets out a similar analysis for the public good. But, there are significant differences. Figure 2 assumes truthful revelation of consumers' preferences for the public good, so that the conventional demand curves can be used for the analysis. Samuelson (1954) refers to $D_{sA}$ and $D_{sB}$ as pseudo demand curves because the assumption of truthful revelation of consumer preferences, strictly speaking is a highly restrictive one in this context. The vertical summation of individual demand curves yields $D_{sA+B}$, the market demand curve for the public good, which is sometimes called the collective willingness to pay curve. The intersection of this curve with the supply curve defines the equilibrium quantity, $Q_{ss}$* in Pigou-Musgrave model as illustrated in Figure 2, which is in excess of the individual demands, i.e. $Q_{ss}^* > q_{sB} > q_{sA}$.

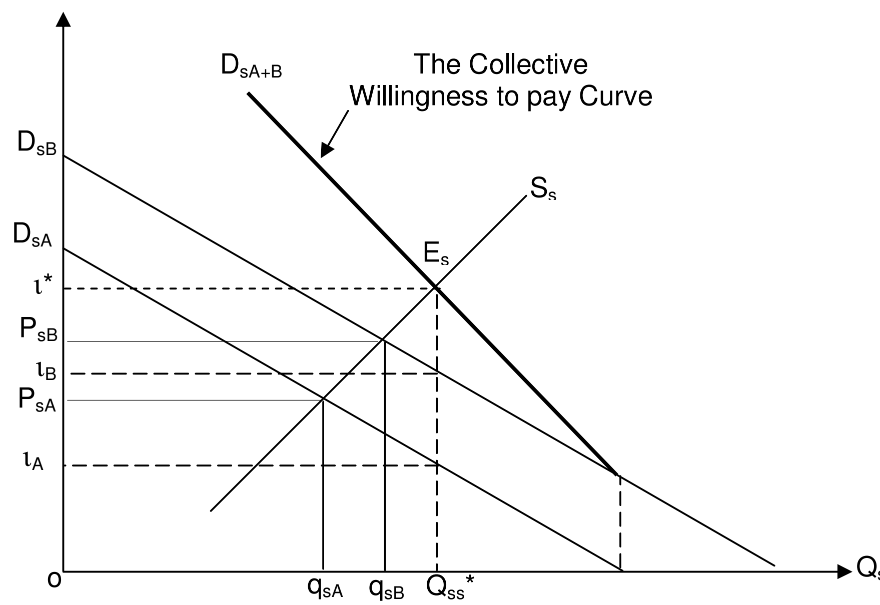


Figure 2: Analysis of the Optimal Supply of Public Good

Figure 2 reveals interesting things about the preferences of individuals A and B, which are distorted by the indivisibility property of the public good, $Q_s$ and their attitudes to this situation.[3] It shows that **if** $Q_s$ were divisible as the private good in Figure 1, individual A would have purchased no more than $q_{sA}$ for which it is willing to pay $P_{sA}$ per unit. Similarly, B would have demanded no more than $q_{sB}$ for which it is willing to pay $P_{sB}$ per unit. The market demand for $Q_s$ therefore would have been $q_{sA}$ + $q_{sB}$ for which A and B would have

[3] Surprisingly, the concept of indivisibility as used here seemed to have caused confusion. It shouldn't. In Figure 1, the divisibility of the private commodity in consumption allowed individuals A and B to buy the quantities each wanted. This is simply not the case with the public good. So, the analogy with private good seems to have been too far.

collectively contributed $P_{sA}q_{sA} + P_{sB}q_{sB}$. This would have been the case if $Q_s$ were divisible in consumption and could be provided with individual exclusion. Unfortunately, this is not the case. Since $Q_s$ is not divisible in consumption, individuals A and B are both forced to consume the entire quantity supplied[4], namely $Q_{sA+B} = Q_{ss}^*$, which is in excess of their respective demands. The larger quantity lowered the marginal utilities derived by each consumer for which they are both unwilling to pay $P_{sA}$ and $P_{sB}$, respectively. In the circumstances A is willing to contribute only $\tau_A < P_{sA}$ and B is willing to pay $\tau_B < P_{sB}$. Figure 2 defines vertical summation as the adding-up mechanism for unit contributions made by all consumers for the total quantity of the public good supplied, namely $Q_{ss}^*$.

Another consequence of the indivisibility of the public good that is quite visible in Figure 2 is that, given the differences in their preferences, individuals A and B derive different marginal utilities for the same quantity of the public good even in equilibrium unlike the case with the private good. For this reason, they are also unwilling to pay the same price (in taxes) for the same quantity of the public good. Further, it is the sum of the individual prices paid by A and B that equal the marginal cost of production, i.e. $(\tau_A + \tau_B) = \tau_s^* = MC$, which is the Samuelson condition for equilibrium or optimal supply of public goods. This rule says that the efficiency condition for the optimal supply of a public good is the sum of the individual prices (marginal benefits or utilities) equals marginal cost. But, the diagram also reveals that the equilibrium for public goods is layered. Indeed, the precondition for the market equilibrium is equilibrium of each and every consumer, which adds up to the market equilibrium. It is this condition that instructs the charging of benefit taxes, i.e. $MB_i = \tau_i$ in equilibrium. Further, it is only by this principle that are individual equilibrium positions consistent with the market equilibrium and conversely.

[4] It is not always the case that individual's consumption of public goods will always equal total supply, since consumption is subject to need, the capacity to consume and greed. So, an individual can abstain from consumption. Thus, equal availability does not necessarily imply equal consumption. Buchanan's example of fire station is also relevant here and implies variability in quality. But variability in quality and/or quantity does not really pose any serious problems for the theory or the analytics because the marginal valuations reflect these.

## The Bowen and Samuelson's Models

These analyses are addressed to general equilibrium models of optimal inter-sectoral allocation of resources to private and public goods production. Their presentation did not utilize the vertical summation device for aggregating demand for public goods. Consequently, these models are not relevant to the focus of this discourse and so are not discussed further in this paper. According to Brown and Jackson (1990), Samuelson's model is a neoclassical generalization of the earlier models of Wicksell and Lindahl and it is these models that are of direct relevance to the focus of this paper.

## Wicksell-Lindahl Approach to Public Goods Supply

This model of analysis unlike the Pigovian model, which emphasized the joint consumption property, emphasized the non-excludability property. However, since non-excludability leads to joint consumption, both properties can be presumed accounted for in this model. But, the model is not founded on the vertical aggregation premise. Their analysis is acutely aware that non-excludability means that A would see B's demand as its supply. Likewise B would consider A's provision as its supply. Therefore, it is improper to aggregate A's and B's demands by whatever means. Accordingly the analysis did not aggregate the demand curves of A and B vertically or otherwise. Given the assumption of truthful preference revelation, their focus was on cooperative production of the public good that will achieve equalized marginal utilities at the same price in equilibrium as is the case with private goods.

Figure 3 provides a geometric interpretation of their model. Costs, cost shares and marginal valuations and hence prices are measured on the vertical axis while quantities of the public good are measured along the horizontal axis. $O_A$ is the origin of individual A. Similarly, $O_B$ is the origin of individual B. The AA curve is the demand curve of A, which B sees as its supply curve. Similarly, the BB curve is the demand curve of B, which A sees as its supply curve. The point of intersection E, equalizes the marginal valuations (marginal utilities) of A and B. Consequently, they are both willing to pay the same price, $\tau^*$ and consume the same quantity, G* of the public good. That is, the total contribution of A and B toward the supply of G* is ($\tau^* + \tau^*$) = $2\tau^*$. It follows that the Wicksell-Lindahl solution recommends a poll or head tax as the financing mechanism. Thus, the Wicksell-Lindahl model is only a special case. Points off E are disequilibrium situations in which A and B insist on different quantities for which they are only willing to pay different prices as exemplified by $G_A$ and $G_B$. Thus, for points off E, the outcome is a priori indeterminate. The point ($\tau^*$, G*) is referred to as **Lindahl equilibrium** in the literature. The beauty of it is that it satisfies unanimity.

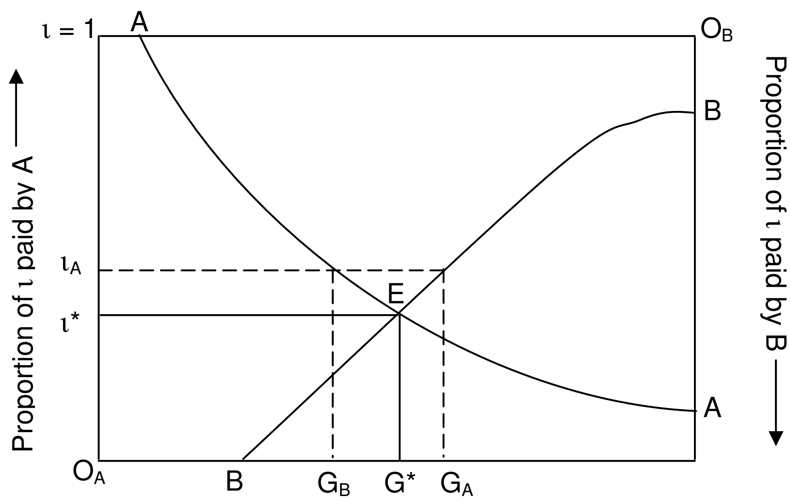


Figure 3: Wicksell-Lindahl Model of Optimal Public Good Supply

Some writers for example Brown and Jackson (1990) regard the Lindahl equilibrium as Nash (competitive) equilibrium. It is not. It is a cooperative equilibrium. How else can the intersection of the two demand curves in equilibrium be interpreted? The intersection of the demand curves of A and B signifies that the choices of these individuals are coordinated into agreement. It is for this reason that E is an equilibrium position. The important point for the purposes of this paper, however, is that the optimal quantity in this model is not derived from a vertical aggregation of the individual demands, even though the individual demand curves are explicitly considered. Neither is it necessary to add up the individual demands vertically nor is this sufficient for the determination of the optimal level of a public good.

Further comments on Figure 2 are in order. While the vertical summation is capable of defining the efficient and equitable cost sharing formula, it exaggerates the optimal demand for a pure public good. Given that pure public goods are non-rival in consumption. For instance, in Figure 2, individual B's demand (provision) over satisfies the demand by individual A, i.e. $q_{sB} > q_{sA}$. So, individual A can comfortably share in what individual B has provided. Consequently, there is no need to add A's demand to that of B in order to determine the optimal quantity. The optimum quantity demanded of it is simply the quantity demanded by individual B, which is $q_{sB}$ and not $Q_{ss}^*$ as the Pigovian model in Musgrave's framework suggests. Thus, the conceptualization and definition of the aggregate demand for public goods in the Pigovian model followed by R. A. Musgrave as illustrated in Figure 2 is not correct. So, for purposes of planning, it is not the aggregate of the individual demands summed vertically that is relevant but the greatest quantity demanded by a single consumer of a public good.

Some examples can clarify the idea and the arguments in its support. Consider the provision of an effective national defence, it is for national honour, but, it protects all residents from foreign conquests. The quantity provided is not arrived at by adding vertically or otherwise of the individual resident's demand for defence. Consider electronic broadcasting, it is simply sent through the airwaves and all those within the coverage area, who care to tune in, receive the messages. Take the highway connecting Gaborone to Francistown (i.e. any two cities anywhere). The length of this highway is determined by the demand for road by people in Francistown to travel from Francistown to Gaborone and back. Once this highway is constructed all the communities in-between these two towns can freely commute to and from each of these cities without requiring their own individual roads. Consider flood control, if the largest demand is a 10-feet high dam or water-brake, once this is constructed, then, all those in the area requiring 9.999-feet to zero inches of dam or water-brake are automatically provided for. This is the point. Whatever quantity that is provided of a public good would be used by all those who demand it under non-rivalrous consumption. **So why aggregate individual demands**? The Lindahl's model seems to have this point of view, even if implicitly. Contrast this to a situation in which ten persons are demanding 1, 2, 3, …, 10 feet of dam, if these demands are added up vertically to determine the optimal quantity of dam (water brake) required, then, the government would build a 55-feet dam, 45 feet of which is a waste.

Indeed, this concept of waste is quite obvious in the Buchanan's model of independent adjustment (see Chapter 2) of Tizio's and Caio's supply of mosquito repelling service, which motivates and orders the adjustments in the trading model, i.e. the existence of unexploited mutual gains in the supply of the public good. This, however, does not lead to a larger outlay on the provision of the public good as he argued. After all, the public service is already in excess supply in the competitive equilibrium of Tizio and Caio because of the unconscious addition of their individual demands. Rather the outcome is a curtailment of the waste in the competitive supply leading to a saving in resources that made possible an increase in the supply of the private good. It has to be realized that the joint income or endowment of Tizio and Caio is fixed and the rise in their real income is from gain in allocative efficiency rather than increase in resources. The waste under reference arises from implicit aggregation of their separate needs for mosquito repelling service, which led to wasteful duplication of supply. This is the meaning assigned to the fallacy of aggregation.

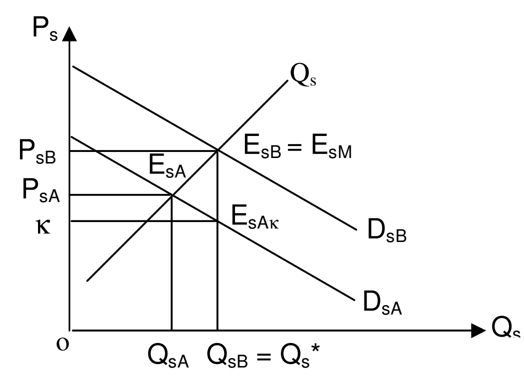


Figure 4: On the Maximum Quantity of Public Goods Demanded, Individual and Market Equilibrium

Figure 4 demonstrates the implications of the viewpoint of this paper. In Figure 4 individual A still demands $q_{sA}$ and is willing to pay $P_{sA}$ per unit for it as in Figure 2 and would be in equilibrium at $E_{sA}$, **if** the public good were divisible in consumption and could be provided with exclusion. However, individual B demands $q_{sB} > q_{sA}$ and is willing to contribute the cost of providing this quantity, i.e. an amount that will cover the total cost under truthful preference revelation. Thus, individual A can safely play the free rider without adversely affecting the supply. Individual B is in equilibrium at $E_{sB}$, which is also the market equilibrium ($E_{sM}$). Given individual A's preferences and $q_{sB}$, if it pays no more than $\kappa$ per unit, individual A is also at equilibrium at both $E_{sA\kappa}$ and $E_{sM}$ simultaneously. Under the assumption of truthful preference revelation, individual A is honest about its attitude to the public good. It admits that the public good in question is useful to it but the larger quantity that it is being forced to consume has reduced its marginal valuation to $\kappa < P_{sA}$, which it is willing to pay per unit. This result is consistent with Henderson and Quandt (1980: 300-2) example of the Lindahl equilibrium and actually generalizes the Lindahl's solution, since in that model the cost shares for the public good provision are not necessarily equal. Indeed, in the two-consumer model that Henderson and Quandt (1980) used to represent the Lindahl model, the cost shares were $\alpha = \frac{4}{27}$ and $(1-\alpha) = \frac{23}{27}$, which lends credence to our conclusion that the Wicksell-Lindahl equilibrium in Figure 3 is a special case.

## Efficient Financing Scheme and Equitable Cost-Sharing Formula

It is clear in Figure 4 that under truthful preference revelation and benefit taxes, public goods would generate budget surpluses, since

$(P_B + \kappa)Q_{sB} > P_{sB}Q_{sB}$ (the total cost of providing $Q_{sB}$) for $\kappa > 0$.

The area of the rectangle $\kappa E_{sA\kappa}Q_{sB}O$ measures the potential budget surplus, which is

$(P_B + \kappa)Q_{sB} - P_{sB}Q_{sB} = \kappa Q_{sB},\ \kappa > 0.$

This says a lot on the fiscal practices that have generated budget deficits the world over in a cumulative manner. An important issue to consider is a higher order implication of benefit taxes, namely, the budget surplus with its potential for fiscal drag on the economy. The model is silent over how this possibility can be handled and also how the cost would be shared efficiently and equitably. These questions, however, can be answered perfectly. The cue lies in the benefit tax principle. The efficient and equitable cost sharing principle in this situation results in a benefit share weighted cost sharing formula. This formula also takes care of the budget surplus and hence the threat of fiscal drag. It says that individuals A and B should pay

$$\frac{\kappa}{(\kappa + P_{sB})}C \text{ and } \frac{P_{sB}}{(\kappa + P_{sB})}C,$$

respectively given that under truthful preference revelation, the individual demand curves represent perfect marginal valuations of the public good such that κ and $P_{sB}$ represent marginal benefits (utilities) to the users – individuals A and B, respectively. That is, the share of the individual in the total benefit determines the proportion of the total cost the individual should contribute or pay toward its provision. Notice that

$$\frac{\kappa}{(\kappa + P_{sB})}C + \frac{P_{sB}}{(K + P_{sB})}C = \text{C}, \qquad (1)$$

which proves that the total cost will be covered by this formula. When equation (1) is compared to Henderson and Quandt (1980) solution to the Lindahl's model, we can decipher the following analogies, i.e. $\frac{\kappa}{(\kappa + P_{sB})}$ = α and $\frac{P_{sB}}{(K + P_{sB})}$ = (1-α).

This result is quite different from the Lindahl's solution, which equalizes the marginal benefits in equilibrium and hence the individual contributions and implicitly or otherwise recommends a poll or head tax. So, it is only a special case of the model espoused in the paper, which can be generalized to cover wide spectrum of cost shares. Given that κ and $P_{sB}$ represent marginal benefits (utilities), the generalized cost sharing formula is

$$\frac{\text{MB}_\text{i}}{\sum_1^\text{m} \text{MB}_\text{i}}\text{C} = \tau_i \text{C (the cost share of individual i)},$$

where C = total cost; $\text{MB}_\text{i}$ = marginal benefit derived by individual i and

$$\sum_{\text{i}=1}^{\text{m}} \text{MB}_\text{i} = \text{total benefit (valuations)}$$

derived by all the consumers of the public good and $\tau_\text{I}$ is benefit share of individual i, which is its unit tax in decimal units with C (the total cost of the public good) as base. This formula, therefore, does not recommend the ability to pay principle of taxation whose base is income.

$$\frac{MB_i}{\sum_1^m MB_i} = \tau_i;$$

is the equilibrium condition for the individual consumers while the condition for market equilibrium is

$$\sum_{i=1}^{m} MB_i = \sum_{i=1}^{m} \tau_i C = C, \Sigma\tau_i; = 1.$$

The model espoused in this paper as illustrated in Figure 4 has a lot of potential for improvements in resource allocation. Comparing the results of this model with those of the vertical summation (Pigovian) model as illustrated in Figure 2, we find that ($Q_{sA+B}$ – $q_{sB}$) measures the excess supply over and above the desirable optimal supply. Eliminating this excess would save resources for other purposes, which could expand both the production and consumption possibility frontiers of the economy. The tax burden is less than the benefits, yet the tax contributions cover total cost. So, there would be no budget deficits or surpluses to worry about. Furthermore, because the tax burden is less than the benefits, taxes would be less resentful to pay. For example, even when B is willing to pay for the entire cost, the willingness of A to share in the cost brings B relief and consumer surplus and/or a budget surplus to the state. It follows that tax evasion and avoidance would be minimized by this benefit weighted cost sharing formula. As far as this model of analysis is concerned, the optimal quantity of a public good to supply is no longer a mystery to providers, the political process and/or the survey approach can easily reveal it.

## Efficient Financing Scheme and Equitable Cost-Sharing Formula

Further comments on the conventional analysis of the optimal supply of public goods are in order. At this juncture the paper is persuaded to invite the profession to give thought reflections on a number of questions that are supposedly closed. For instance, what do we really consider as the marginal cost of a pure public good? Given that it is not possible to produce one half or any fraction thereof (say of a stadium or a bridge) though any number of stadia or bridges can be constructed in principle, what constitutes the marginal cost of a given stadium or bridge? Is the construction of a network of stadia in space and time not belong more in a multi-plant paradigm with implicit implication of the concept of optimal population than within marginal analysis? The Samuelson's condition for the determination of the optimal quantity is

$\Sigma_1^m MB_i = MC$.

What is the interpretation of this condition in the light of the above questions and the fact that the additional user could use a public good without additional cost? It is tempting to suggest for a road project, (say a highway joining Washington, D. C. and New York), that MC can be computed for the next kilometer of the road. But, a little contemplation shows that this corresponds more to average than marginal cost. At any rate, a kilometer of a road is only a fraction of the road and since a fragment cannot be the whole, a fragment of a product cannot be the product and so cannot be the basis for computing MC. How does the concept of the marginal cost of producing a public good compare with that of producing a box of matches or a packet of biscuits or a deep freezer or a DVD set? What is it and how should it seen?

It has been said that the indivisibility of public goods in production says nothing about the details of the cost function? But, what is the cost function? Is it not the dual of the production function?[5] Or is this a misinformation to students by the profession? The above questions suggest that the marginal cost of a pure public good is zero. Therefore, the Samuelson condition reduces to

$\Sigma_1^m MB_i = MC = 0$.

It follows that this optimal condition is actually a recommendation for the debasement of scarce resources. It also means that the marginal cost curve for the supply of public goods does not exist just as the demand curve for public goods. It is intriguing that Figure 3 determined the optimal quantity without reference to marginal cost or supply curve. So, the

[5] See Varian (1978, 1980) Microeconomic Analysis.

Pigovian analysis and all other analyses of public goods supply based on the marginal cost curve crumbles completely. Some writers for example McConnell and Brue (1998) even talk about the last unit of the public good. A fundamental question is whether or not such a quantity exists? After all, the indivisibility property of public goods is even stronger in production than in consumption, at least for some public goods. Thus, the supply curve for a public good is the average cost curve, which is upward sloping for the well known reasons, though, it could be horizontal or even downward sloping but would never vanish. Therefore, the suggested general optimal condition for public goods supply is

$\Sigma_1^m MB_i = AC$.

For some people, this condition might raise questions about or issues of Pareto optimality from the perspective of the orthodox definition. Such questions, however, are misconceived and misplaced because they fail to appreciate that equilibrium or the optimum is relative to the reality of situations. This in no way debates that Pareto optimal solutions are the first bests. But, a pertinent question is whether Pareto optimal solutions are always feasible? If the answer to this question is in the negative, then second best solutions are not trivial. Of course, the Samuelson condition holds and is appropriate, if a given public good has a well defined and continuous marginal cost function.

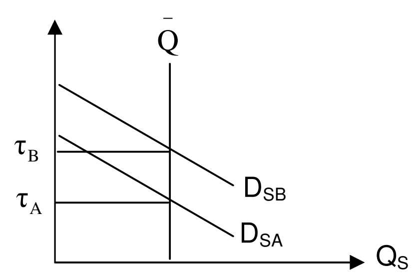


Figure 5: A Model of Fixed Public Goods Supply

What seems to be stirring us in the face is a fixed supply scenario. A 100,000-seater capacity stadium or a four-lane bridge specified to carry set load is fixed in supply. It is of course understood that in time facilities can be expanded. But, this sounds more of scale dimensions than marginal adjustment. This perspective leads and is suggestive that the appropriate analytical framework is that illustrated in Figure 5. This vertical supply curve shifts outward to the right when facilities are expanded and to the left if facilities are left to rot and decay. This model thus seems to describe reality much more closely than marginal analysis. It is intriguing that the results in Figure 5 conform and further strengthen the results obtained in Figure 4.

## Summary and Concluding Remarks

This paper revisited the analysis of optimal public goods supply. It argues that the vertical summation of individual demands for a public good as in the Pigovian model as sold by Musgrave is an improper framework for the determination of the optimal quantity of a public good to supply. Specifically, this method is accused of overestimating the desirable optimal quantity of a public good. The paper submits that the optimal quantity for a public good is the largest quantity demanded by any single consumer (individually or as a collective) per unit time. A corollary of this is that public goods consumption is not validly subject to aggregation by any methods including vertical summation. Aggregation is irrelevant and that the individual demand curves or schedules are required only for the determination of optimal benefit taxes and equitable cost sharing formula. That is, each person's consumption or utilization of the good or service must be considered separately only for the purpose of determining each person's fair and equitable share of the cost of supply based on its marginal benefit. So, the paper recommends the use of the model illustrated in Figure 4 instead of the Pigou-Musgrave model as illustrated in Figure 2. It further thinks that Figure 5 provides a more realistic analytical framework for the supply of public goods than any other voluntary exchange model.

It is found that barring information failure, the public budget should ordinarily be enjoying surplus under optimal benefit taxes. Otherwise, balanced budget is within easy reach with relief for taxpayers in relation to benefits enjoyed. Therefore, less resentment to taxation could be anticipated. An efficient financing scheme and equitable cost sharing formula is also proposed. It is a benefit share weighted cost sharing formula, namely $\Sigma\alpha_i P_s$ =

$P_s$, where $\Sigma\alpha_I = 1$; $P_s$ = the price of the public good and $\alpha_i$ is the benefit share of individual i as defined in equation (1).

Reckoning with the nature of public goods, the paper notes that the marginal cost of providing public goods to additional users is zero. It also believes that marginal cost curves for public goods do not exist in the orthodox sense. So, marginal analyses are inappropriate for the supply of public goods. This means that all analyses of optimal public goods supply based on the marginal cost curve are null and void. Interpreting the Samuelson's condition for optimal public good supply in this light, the paper concludes that it is a recommendation for the debasement of scarce resources, which does not make any good economic sense. In its place, the paper recommended the equality of the sum of marginal benefits (utilities) to the average cost of production as the condition for the optimal supply of public goods. It also concluded that the relevant supply curve for public goods is the average cost curve, yet, it believes that the supply curve of many a public good such as roads and bridges is more of a vertical line denoting fixed supply than anything else.

## BIBLIOGRAPHY

Agiobenebo, T. J. (2003), Public Sector Economics: Theories, Issues and Applications, Vol. 1 (2nd ed., Lima Computers, Port Harcourt

Brown C. V. & P. M. Jackson (1990), Public Sector Economics (4th Edition, Basil Blackwell, Oxford, UK)

Buchanan, J. M. (1968), The Demand and Supply of Public Goods (Collected Works)

Henderson, J. M. & R. E. Quandt (1980), Microeconomic Theory: A Mathematical Approach (3rd ed., McGraw-Hill International Editions, London)

McConnel, C. R. & Stanley L. Brue (1998), Economics (International Edition, Irwin MacGraw-Hill, Boston, USA)

Musgrave, R. and P. Musgrave (1989), Public Finance in Theory and Practice (8th ed. McGraw-Hill International Editions, McGraw-Hill Book Company, N.Y.)

Pigou, A. C. (1928), A Study in Public Finance (Macmillan, London)

Samuelson, P. A. (1954), "The Pure Theory of Public Expenditure." Review of Economics and Statistics, Vol. 36